Homework 9: Complexity Theory

Due: December 5, 2024

Problem 1. Given an undirected graph $G = (V, E)$ and a subset of its vertices V', the sub-graph induced by V' is defined as $G' = (V', E')$ where E' includes all edges from E with both endpoints in V' (that is $E' = (V' \times V') \cap E$).

A set $V'' \subseteq V$ is a *vertex cover* of G if all edges in E have at least one endpoint in V''. The *size* of a vertex cover is the number of vertices in it. We say that V'' is a *connected vertex cover* if the subgraph induced by V'' is connected.

The "k-connected vertex cover problem" (k-CONCOV) is a decision problem for which, given as input an undirected graph G and a positive integer value k, we want to decide whether there exists a connected vertex cover of G of size k or less.

(a) Present a deterministic algorithm for solving k-CONCOV. Your algorithm should run in $O\left(n^{k+2}\right)$ worst-case time, where n is the number of vertices in the graph. Argue the correctness of your algorithm and analyze its running time.

(b) Prove that k-CONCOV $\in NP$.

Solution. □

Problem 2. Consider the languages:

$$\begin{aligned}\mathsf{TFSAT}_k = \{\phi \mid \phi \text{ is a Boolean formula in } \mathsf{CNF} \text{ with k elements in each clause.}\\ \text{There exists an assignment such that in each clause,}\\ \text{there is at least one true and one false literal}\}\end{aligned}$$

Show that $TFSAT_4$ is NP-complete.

Solution. □

Problem 3. Consider the languages:

$\mathsf{EX1SAT} = \{\phi \mid \phi \text{ is a satisfiable Boolean formula in } \mathsf{CNF} \text{ with exactly 1 literal true per clause}\}$

Show that $EX1SAT$ is NP-complete.

Solution. □