Homework 8: Pattern Matching & Computational Geometry

Due: November 22, 2024

Problem 1. In the fuzzy pattern-matching problem given a text T and a pattern P constructed using symbols from an alphabet Σ, we are looking to verify if the pattern P appears in the text T. However, we are accepting as valid matches occurrences of P in T for which, at most, one character is mismatched. E.g, let $T = aaaaaabd$ and $P = abc$, there is a fuzzy match of P in T for abd with at most one mismatch.

1. Show how to modify the Rabin-Karp algorithm for this problem:

   - The initialization should require almost at most $c_1|\Sigma||P|$ time where c is a constant value with respect to $|P|$, T, and $|\Sigma|$.
   - *Besides for the initialization phase*, the algorithm should run in the expected time $c|T|$, where c is a constant values with respect to $|P|$, T, and $|\Sigma|$. You can assume a constant number of collisions with high probability.

   Overall, the expected running time of your algorithm should be $\mathcal{O}\left(|\Sigma||P| + |T|\right)$. NOTE: You cannot assume $|\Sigma|$ to be constant with respect to $|P|$ and $|T|$.

2. Prove the correctness of your algorithm.

3. Analyze the worst-case running time of your algorithm

Solution. □

Problem 2. Lorenzo is mixing paint for his house. Since his favorite color is green he's mixing some blue paint and yellow paint together with some paint thinner. He's managed to create n different shades of green with all the blue and yellow paint that he has.

For example, suppose he made three different shades of green.

		Samples		
% Compound		S_1	S_2	S_3
	Yellow	0.7	0.3	0.1
	Blue	0.2	0.1	0.7
	Paint Thinner	0.1	0.6	0.2

Then it is possible to produce a shade of green that is 35% yellow and 27.5% blue by mixing the shades he currently has in a $1 : 2 : 1$ ratio (25% S_1, 50% S_2, 25% S_3). However, it is **impossible** to create the shade of green which is 20% yellow and 10% blue.

Design an $O(n \log n)$ algorithm that checks whether it's possible to create a liquid with the specified percentage of yellow and blue. Argue the correctness of your algorithm.

Example Input: [(0.7, 0.2), (0.3, 0.1), (0.1, 0.7)], (0.35, 0.275)
Output: True

Hint: What is this nonsense about paint colors? I wonder! I guess possible ratios of Yellow and Blue used to obtain the n shades of green look like coordinates of points on the plane... :)

Solution. □

Problem 3. Aditya is building a fence for his farm to protect against the invading squirrels. He plans on using fence posts with positions given by $Q = [q_0, q_1, \ldots, q_{n-1}]$, which are sorted in counterclockwise order with respect to q_0. Every fence post should be used as part of the resulting fence or contained entirely within it. To ensure he could always go to and from any place on his farm at the minimum distance possible, he constructed his fence in the shape of a convex polygon.

A year after building the fence, Aditya wants to add in a new fence post with position p. Describe a linear time algorithm that Aditya can use to modify his fence to include p (either as part of the fence or contained entirely within it) so that it still contains all the fence posts in Q. Show that your proposed algorithm is correct and analyze its running time.

Solution. □